

## **SIMULATION TESTS OF THE METHOD FOR DETERMINING A CAD MODEL OF FREE-FORM SURFACE DETERMINISTIC DEVIATIONS**

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### **Abstract**

Geometric deviations of free-form surfaces are attributed to many phenomena that occur during machining, both systematic (deterministic) and random in character. Measurements of free-form surfaces are performed with the use of numerically controlled CMMs on the basis of a CAD model, which results in obtaining coordinates of discrete measurement points. The spatial coordinates assigned at each measurement point include both a deterministic component and a random component at different proportions. The deterministic component of deviations is in fact the systematic component of processing errors, which is repetitive in nature. A CAD representation of deterministic geometric deviations might constitute the basis for completing a number of tasks connected with measurement and processing of free-form surfaces. The paper presents the results of testing a methodology of determining CAD models by estimating deterministic geometric deviations. The research was performed on simulated deviations superimposed on the CAD model of a nominal surface. Regression analysis, an iterative procedure, spatial statistics methods, and NURBS modelling were used for establishing the model.

Keywords: coordinate measurement, geometric deviation, free-form surface, regression analysis, CAD model.

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### **1. Introduction**

Free-form surfaces are characterized by spatially complex geometry. In designing, producing, and inspecting the accuracy of such surfaces, CAD/CAM techniques and numerically controlled machines are used. Processing is carried out with multi-axis milling machine centres. Accuracy inspection is most often performed using numerically controlled CMMs (coordinate measuring machines) equipped with touch measurement probes. The result of a measurement is a set of measurement points of a specified distribution on the measured surface. For each measurement point, the value of the local geometric deviation, i.e. the distance of this measurement point from the CAD model of the nominal surface in the normal direction is established. Measurements can be made with reference to the datum features, and then the measurement results include also deviations of location and orientation. In order to reduce deviations in location and orientation, fitting the measurement data to the CAD model needs to be performed; if it is the case, then local geometric deviations only represent surface irregularities and it is possible to assess a (simple) form deviation from them [1].

Because the curvature is spatially variable at each point of a free-form surface, the distribution of machining forces and other phenomena occurring during processing are also spatially variable [2, 3]. As an effect of this, the distribution of geometric deviations is of the same character. Deviations of a systematic (deterministic) character, i.e. deviations which are repeated on subsequent surfaces processed under the same conditions, as well as deviations of

a random character, are observed on a surface. Deterministic deviations are spatially correlated, however a lack of spatial correlation indicates spatial randomness of deviations [4]. Spatial statistics methods are applied to research on dependency of spatial data [5-8] (Section 2.2).

Identifying spatial autocorrelation of local geometric deviations proves the existence of a systematic (deterministic) deviation. In such a case, an advanced CAD software for surface modeling may be applied to fit a surface regression model estimating the systematic deviations. The first step in model diagnosing is to examine the model residuals for the probability distribution and the existence of spatial autocorrelation [5-7].

The paper presents the results of testing a methodology of determining CAD models representing systematic geometric deviations. The research was performed on simulated deviations superimposed on the CAD model of the nominal surface. Regression analysis, an iterative procedure, spatial statistics methods, and NURBS modelling [9] were used for establishing the model. The patch surface interpolation and shape modification were performed with the use of Rhinoceros software which is a geometric modeler based on the NURBS method.

## **2. The CAD model of deviations**

Geometric deviations of surfaces are attributed to many phenomena that occur during machining, both deterministic and random in character. These phenomena with their consequent machining errors can be described in the space domain. In coordinate measurements of free-form surfaces, spatial data is obtained which provide information on the processing and on geometric deviations in the spatial aspect. If a measurement is to take into consideration form deviation without reference to the datum features, the procedure of fitting the data to the CAD model must be performed [1, 4]. Then, the determined local geometric deviations (Fig. 1) only represent surface irregularities which can be divided into three components of different length: form deviations, waviness, and roughness of the surface [10]. Spatial coordinates of each measurement point include all the three components in different proportions [4, 8].

The spatial coordinates assigned to each measurement point include two separate components, deterministic and random [10]. The component connected with the deterministic deviations is spatially correlated. The random component, on the other hand, is weakly correlated and is considered to be of a spatially random character [4, 8]. Deviations of random values may be spatially correlated which is reflected in their deterministic distribution on a surface and is indicative of the existence of a systematic source in the course of processing. Lack of spatial correlation indicates spatial randomness of deviations. The different nature of geometric deviations may be the basis for removing the random component from measurement data [4-6].

If a test for spatial autocorrelation shows spatial dependence of the data, a spatial model representing deterministic deviations can be prepared [5, 6]. With reference to free-form surfaces which are machined and measured in a CAD environment, a CAD model of deviations, presented in one of the standard formats of data exchange (e.g. IGES), has a practical use in correcting machining errors as well as in planning and performing measurements.

## 2.1. Modeling the surface of deterministic deviations

In order to create the surface model estimating deterministic deviations, the NURBS method was applied. The NURBS surface of the  $p$  degree in the  $u$  direction and the  $q$  degree in the  $v$  direction is a vector function of two variables in the form of [9]:

$$S(u, v) = \frac{\sum_{i=0}^n \sum_{j=0}^m w_{i,j} N_{i,p}(u) N_{j,q}(v) P_{i,j}}{\sum_{i=0}^n \sum_{j=0}^m w_{i,j} N_{i,p}(u) N_{j,q}(v)} \quad (1)$$

$P_{i,j}$  points make up a two-direction control points grid on which the surface patch is lofted ( $n, m$  are the numbers of control points in the  $u$  and  $v$  directions respectively),  $w_{i,j}$  are the weights, while  $N_{i,p}(u)$  and  $N_{j,q}(v)$  are the  $B$ -spline basis functions defined on knot vectors [8]. The input data in surface interpolation is a set of points forming a spatial grid of points. In the case under concern, the data were obtained from coordinate measurements during which a two-direction grid of measurement points was obtained. In developing the geometric model, the method of global surface approximation was used. The process is carried out in two stages [9]:

- at the first stage, a series of isoparametric curves located on the surface patch is created. These curves are approximated on the subsequent rows of the pre-set points of one of the parameterization directions,  $u$  or  $v$ ; the value of the other parameter describing the surface is then constant. A spatial grid of control points is obtained in this way;
- at the second stage, coordinates of surface control points are determined. It is performed by approximating curves through the control points of the curves which were approximated earlier. The approximation is made in the other parameterization direction. The surface is lofted on the series of curves which was determined earlier.

After the approximation stage was completed, shape modification iteration of the created surface patch was applied in the subsequent stages. These operations are aimed at obtaining an adequate model of the regression surface which would represent deterministic deviations [4, 8]. In this case, popular procedures were applied of changing the NURBS surface shape, namely [9]:

- rebuilding the knot vectors, which influences a change in the number of control points in the  $u$  and  $v$  directions),
- changing the degrees of  $B$ -spline base functions.

Increasing the number of knots (at the subsequent iteration stages) results in increasing the number of control points of the surface. A more complex shape can be obtained this way.

## 2.2. Models' adequacy verification

The method of adequate model designing consists in iterative modeling of the surface regression model and in testing the spatial randomness of the model residuals at the consecutive iteration stages. In the subsequently constructed models, the number of control points and the surface degrees in both directions are changed [4, 9]. The model residuals are examined each step, and the maximum and minimum values, arithmetic mean (should be  $\sim 0$ ), probability distribution (the distribution normality was verified with the Kolmogorov-Smirnov test), and the  $I$  spatial autocorrelation coefficient (2) are determined.

The spatial autocorrelation coefficient  $I$  has the following form [5]:

$$I = \frac{n}{S_0} \frac{r^T C r}{r^T r}, \quad (2)$$

where:

$r$  – vector of model residuals,  $C$  – weighting matrix of spatial relations between residuals in  $i$  and  $j$  locations,

$$S_0 = \sum_{i=1}^n \sum_{j=1}^n c_{ij}, \quad (i \neq j). \quad (3)$$

It is assumed that the dependence between the data values at the  $i$  and  $j$  points decreases when the distance increases, this relation can be described in the following way:

$$c_{ij} = d_{ij}^{-f}, \quad (4)$$

where:

$c_{ij} = 0$  for  $i = j$ ,  $f$  – constant ( $f \geq 1$ ), in this work  $f = 3$  is assumed [8].

After having determined the coefficient  $I$ , a test of significance of its value needs to be conducted. A positive and significant value of the  $I$  statistics implies the existence of positive spatial autocorrelation, i.e. a similarity of residuals in the specified  $d_{ij}$  distance. Otherwise, lack of spatial correlation indicates spatial randomness of residuals [5, 6, 8].

The model with the smallest number of control points and the lowest surface degrees in the  $u$  and  $v$  directions (Section 2.1), for which the model residuals met the criteria of a normal distribution and of spatial randomness, is adopted as an adequate one.

### 3. Simulation tests

The performed simulation tests were aimed at verifying the methodology of developing a model of a surface which would estimate deterministic deviations (Section 2) and whose key element was the adopted structure of the spatial weights matrix. The tests consisted in modeling the regression surface on the simulated geometric deviations including both the deterministic component and the random component with pre-set values and spatial distribution, followed by testing the deviations obtained from the model. This method was assessed by comparing the sets of deviations obtained as a result of applying the methodology (the output data) to the simulated data (the input data).

#### 3.1. Simulations of geometric deviations

The cutter deflection is the most dominant factor leading to geometric deviations on free-form surfaces. The model for generating surfaces in free-form machining, proposed by Lim and Menq in [2], was applied to simulate deterministic deviations caused by the cutter deflection:

$$d = \frac{SF}{k} = \frac{F \sin \varphi}{k}, \quad (5)$$

where:

$S$  – deflection sensitivity at the surface nominal point along the horizontal direction, equal to approximately  $\sin \varphi$ ,  $F$  – the horizontal cutting force applying to the cutter,  $k$  – the stiffness of the cutter at the surface generation point,  $\varphi$  – the angle between the tool axis and the surface normal vector at the surface generation point.

2500 points located along a regular  $u \times v$  grid were generated on a CAD model of a surface (Fig. 1) with the base measuring  $50 \times 50$  mm.

Afterwards, the deviations determined according to the dependence (5) as well as deviations of random values were superimposed at the obtained points; the scatter intervals were the same for both of the sets. The maps of the simulated deviations are shown in Fig. 2 and Fig. 3a. In this way, local deviations containing a deterministic component and a random component, i.e. deviations of a mixed character, were obtained (Fig. 4).

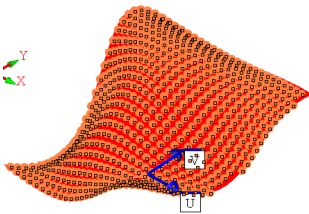


Fig. 1. CAD model of the surface with generated points.

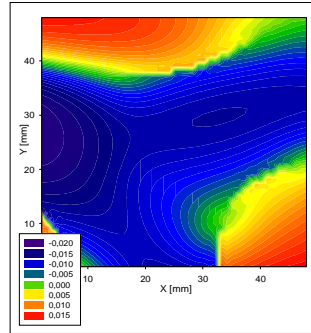
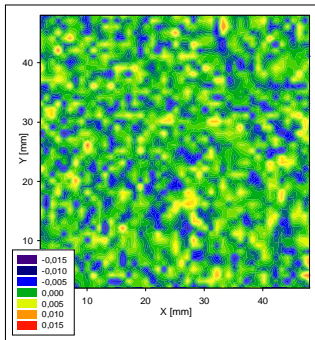


Fig. 2. The map of simulated deterministic deviations.

a)



b)

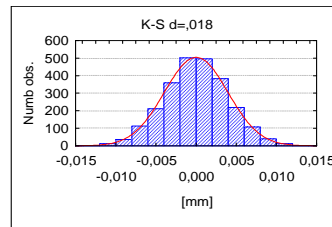


Fig. 3. The map (a) and probability distribution (b) of simulated random deviations.

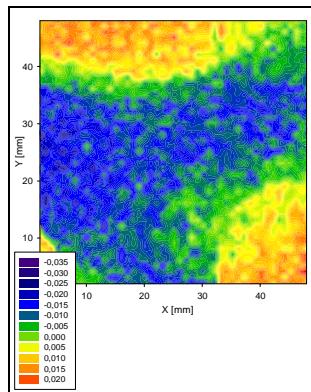


Fig. 4. The map of simulated deviations.

For both data sets, tests on probability distribution and spatial autocorrelation of geometric deviations were subsequently carried out. The procedure described in Section 2 was applied, and in all statistical tests a confidence level  $P = 0.99$  was adopted. The significant features of the sets of simulated deviations were analysed quantitatively. The statistical parameters of simulated deviations are compiled in Table 1. The test results confirmed the assumptions on the spatial (random, deterministic, and mixed) character of the simulated deviations. This means that the adopted test method detects spatial dependence between deviation values, and that, according to this method, a set of mixed deviations represents the actual situation in which the effects of systematic and random processing errors appear on a surface.

Table 1. Statistical parameters of simulated deviation sets.

	Variants of simulated deviations		
	deterministic	random	mixed
Min. [mm]	-0.0180	-0.0147	-0.0321
Max. [mm]	+0.0156	+0.0150	+0.0223
Mean [mm]	-0.0054	0.0000	-0.0054
Standard dev. [mm]	0.0096	0.0039	0.0103
K-S $d$ statistics ( $d_\alpha = 0.033$ )	0.173	0.018	0.105
Correlation coefficient $I_p$	0.937	0.023	0.806
Test statistics $z$ ( $z_\alpha = 2.34$ )	86.183	2.188	74.144

### 3.2. Determining the model of deviations

In the second stage, regression surface estimating systematic deviations were modeled on the obtained data using an iterative procedure, NURBS geometric modeling, and spatial statistics methods (Sections 2.1, 2.2). The model with the smallest number of control points and the lowest surface degrees in the  $u$  and  $v$  directions, for which the model residuals met the criteria of a normal distribution and of spatial randomness, was adopted as an adequate one. In this case the criterion was met for the number of control points amounting to  $28 \times 28$ , and the number of surface degrees being  $2 \times 2$ . The modelling and computation results are compiled in Table 2. The determined CAD model represents deterministic deviations (Fig. 5), whereas the residuals of the model (Fig. 6) constitute the random deviations.

Table 2. Modelling and computations results.

	Statistical parameters	Modelling
Systematic comp.	Number of control points	$28 \times 28$
	Model degrees in $u \times v$ directions	$2 \times 2$
	Min. ÷ Max. [mm]	$-0.0250 \div +0.0219$
	Mean [mm]	-0.0054
	Standard deviation [mm]	0.0100
	K-S statistics ( $d_\alpha = 0.033$ )	0.126
Random comp.	Min. ÷ Max. [mm]	$-0.0138 \div +0.0148$
	Mean [mm]	0.0000
	Standard deviation [mm]	0.0038
	K-S statistics ( $d_\alpha = 0.033$ )	0.027
	Correlation coefficient $I$	0.016
	Test statistics $z$ ( $z_\alpha = 2.34$ )	2.071

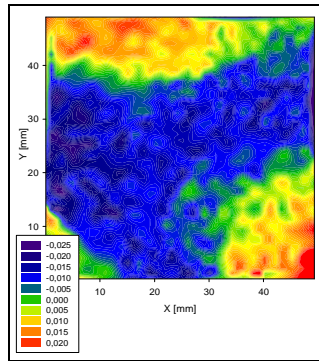


Fig. 5. The map of deviations represented in a CAD model of deterministic deviations.

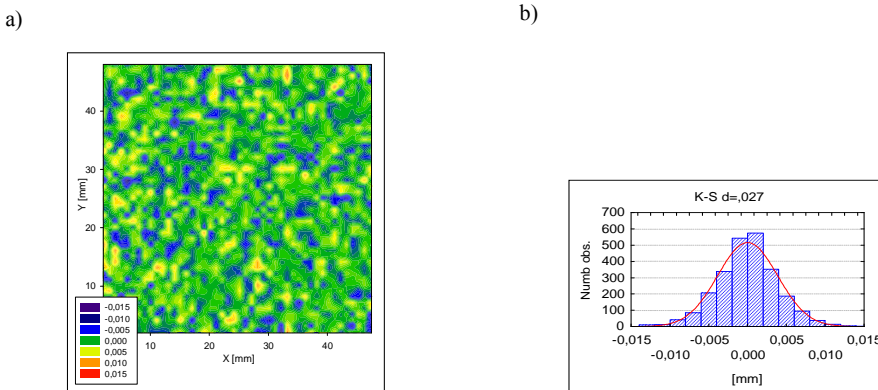


Fig. 6. The map (a) and probability distribution (b) of model residuals.

As the calculation results show (Table 1, Table 2), the arithmetic mean values for the two deterministic components, the simulated one and the one determined from the regression model, were equal, and the obtained standard deviation values were very similar. Analogous conclusions on the similarity of the statistical parameters can be drawn with regard to the random component of each set. As it can be seen, the deviations forming clusters of values (Fig. 3a) whose sizes are detected in tests for spatial autocorrelation, were separated from the simulated random deviations as a result of the applied procedure (Fig. 6a). In consequence, the random model residuals were characterized by lower values of their spatial autocorrelation coefficient (Table 2). The scatter ranges of the values and of the deterministic components were significantly different – the scatter of the deterministic component obtained from the regression model was greater. Besides the simulated deterministic component, the spatial regression model presented in Fig. 5 contains an (added) part of the simulated random component with random values and a spatially deterministic distribution. The obtained bigger scatter is the result of summing the simulated deterministic component and the separated part of the simulated random component.

In the performed simulation tests, it could be seen as the result of the accomplished analysis that a part of the simulated random component characterised by random values but also by a spatially deterministic distribution on the surface, was contained in the CAD model

representing deterministic deviations (compare Fig. 5 and Fig. 2). The remaining part, with random values and with a random spatial distribution, was separated from the simulated deviations (compare Fig. 6 and Fig. 3).

#### 4. Conclusions

The generated data simulated local geometric deviations representing surface irregularities of a mixed character. As the result of applying a procedure of analyzing geometric deviations, a spatial surface CAD model was obtained which describes deterministic deviations as well as a set of local deviations which are random considering their values and their spatial distribution. The local deterministic deviations, determined from the regression model, included the simulated deterministic component and also a part of the simulated random component characterized by a deterministic spatial distribution. The tested procedure takes account of the randomness of both the values and the spatial distribution of the local deviations. The method makes it possible to reject deviations of a random character from the measurement data set. The established regression model, representing systematic deviations, contains a component which is of a deterministic character both with respect to its values and to its spatial distribution. On the basis of the obtained model, conclusions can be made on processing errors of a systematic character. The model described above is obtained in digital form; it can be exported in any standard format, e.g. IGES, and then used for correcting the processing program or planning and performing accuracy inspection of surfaces processed under the same conditions.

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